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Renormalization Group Evolution in the type I + II seesaw model

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We carefully analyze the renormalization group equations in the type I + II seesaw scenario in the extended standard model (SM) and minimal supersymmetric standard model (MSSM). Furthermore, we present analytic formulae of the mixing angles and phases and discuss the RG effect on the different mixing parameters in the type II seesaw scenario. The renormalization group equations of the angles are independent of the Majorana phases. In addition, there is a contribution which is proportional to the mass squared difference for a hierarchical spectrum. This is in contrast to the inverse proportionality to the mass squared difference in the effective field theory case.

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I. INTRODUCTION

In the last years, the precision of the leptonic mixing parameters has been further increased [1] due to a number of different experiments [2, 3, 4, 5, 6, 7]. The precision of measurements of leptonic mixing parameters will even increase in upcoming experiments. In the next-generation experiments, the mixing parameters will be measured on a 10 % level [8]. Thus we are entering an era of precision experiments in neutrino physics.

Contrarily, there have been a lot of attempts to explain the structure of the neutrino mass matrix (*e.g.* [9, 10, 11]). However, most models use heavy particles which generate light neutrino masses effectively after decoupling. Examples are right–handed neutrinos in the standard (type I) seesaw mechanism [12, 13, 14, 15, 16]

$$m_\nu \,=\, -(m_\nu^{\mathrm{Dirac}})^T\, M^{-1}\, m_\nu^{\mathrm{Dirac}} \tag{1}$$

or a Higgs triplet in the type II seesaw mechanism [16, 17, 18]

$$m_\nu = v_\Delta Y_\Delta \ , \tag{2}$$

where v_Δ is the vacuum expectation value (vev) of the Higgs triplet and Y_Δ is the Yukawa coupling matrix of the vertex $\ell\Delta\ell$. Clearly, the seesaw operates at high energy scales while its implications are measured by experiments at low scales. Therefore, the neutrino masses given by Eqs. (1,2) are subject to quantum corrections, i.e. they are modified by renormalization group (RG) running.

The running of neutrino masses and leptonic mixing angles has been studied intensely in the literature. RG effects can be very large for a quasi-degenerate neutrino mass hierarchy and they can have interesting implications for model building. For instance, leptonic mixing angles can be magnified [19, 20, 21, 22, 23], the small mass splittings can be generated from exactly degenerate light neutrinos [24, 25, 26, 27, 28, 29] or bimaximal mixing at high energy can be made compatible with low-energy experiments [30, 31, 32]. On the other hand, even rather small RG corrections are important in view of the precision era we are entering. For example, RG effects induce deviations from $\theta_{13} = 0$ or maximal mixing $\theta_{23} = \pi/4$ [33, 34, 35] also for a hierarchical spectrum, as well as from other symmetries, like quark-lepton-complementarity [36, 37, 38, 39], tribimaximal mixing [39, 40, 41, 42, 43, 44] and other special configurations [45, 46]. Threshold corrections can yield large effects [21, 30, 31, 32, 34, 38, 43, 47, 48, 49, 50, 51, 52].

These studies have been done in the effective theory of Majorana neutrino masses [33, 34, 35, 53, 54, 55, 56]. There are also studies in the standard seesaw case [52, 57] and in the case of Dirac neutrinos [58, 59] which also show significant RG effects which can become comparable to the precision of experimental data. Therefore the RG effects have to be considered in model building in order to be able to compare predictions to experimental data.

In this paper we present the RG equations in the type II seesaw scenario [75] in the standard model (SM) [60] and minimal supersymmetric standard model (MSSM). We derive analytic formulae which allow to understand the running of the neutrino parameters above the threshold of the Higgs triplet. Furthermore, we extend the software package REAP/MixingParameterTools [76] by an Higgs triplet for analyzing the RG evolution numerically. A similar calculation has been done by Chao and Zhang on the renormalization of the SM extended by a Higgs triplet [61]. The two calculations differ in several terms [77].

The paper is organized as follows: In Sec. II, we present the Lagrangian of the type II seesaw model and give the tree–level matching conditions. Furthermore in Sec. III we show all new wave function renormalization factors and counterterms and point out the differences to the work of Chao and Zhang [61]. The RG equations are shown in Sec. IV. In Sec. V, the additional terms in the superpotential and the RG equations in the MSSM are presented[78]. Sec. VI is dedicated to the analytic understanding of RG effects in the type II seesaw case (only a Higgs triplet) and Sec. VII gives a glimpse on the full type I + II seesaw case. Finally, Sec. VIII contains our conclusions.

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II. TYPE II SEESAW LAGRANGIAN

In the following, we consider the SM extended by right-handed neutrinos $\nu_R \sim (\mathbf{1},\mathbf{0})_{\mathrm{SU}(2)\times\mathrm{U}(1)}$ and a charged Higgs triplet $\Delta \sim (\mathbf{3},\mathbf{1})_{\mathrm{SU}(2)\times\mathrm{U}(1)}$,

$$\Delta = \frac{\sigma^i}{\sqrt{2}}\Delta_i = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} . \tag{3}$$

The Lagrangian is given by

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_{\nu_R} + \mathscr{L}_{\Delta} , \tag{4}$$

where the individual parts are defined by

$$\mathscr{L}_{\nu_R} = \overline{\nu_R}\partial\!\!\!/\nu_R - (Y_\nu)_{ij}\,\overline{\nu_R}^i \ell_L^j \phi^{\mathrm{C}} - \frac{1}{2}M_{ij}\overline{\nu_R{}^{\mathrm{C}}}^i \nu_R^j + \mathrm{h.c.} \tag{5a}$$

$$\begin{aligned}\mathscr{L}_{\Delta} =& \mathrm{tr}\left[(D_\mu\Delta)^\dagger D^\mu\Delta\right] - \mathscr{V}(\Delta,\phi) \\ & - \frac{1}{\sqrt{2}}(Y_\Delta)_{fg}\,\ell_L^{Tf}\mathrm{C}i\sigma_2\Delta\ell_L^g + \mathrm{h.c.}\end{aligned} \tag{5b}$$

$$\begin{aligned}\mathscr{V}(\Delta,\phi) =& M_\Delta^2\,\mathrm{tr}\left(\Delta^\dagger\Delta\right) + \frac{\Lambda_1}{2}\left(\mathrm{tr}\,\Delta^\dagger\Delta\right)^2 \\ & + \frac{\Lambda_2}{2}\left[\left(\mathrm{tr}\,\Delta^\dagger\Delta\right)^2 - \mathrm{tr}\left(\Delta^\dagger\Delta\Delta^\dagger\Delta\right)\right] \\ & + \Lambda_4\phi^\dagger\phi\,\mathrm{tr}\left(\Delta^\dagger\Delta\right) + \Lambda_5\phi^\dagger\left[\Delta^\dagger,\Delta\right]\phi \\ & + \left[\frac{\Lambda_6}{\sqrt{2}}\phi^T i\sigma_2\Delta^\dagger\phi + \mathrm{h.c.}\right]\end{aligned} \tag{5c}$$

The covariant derivative of the Higgs triplet is given by [79]

$$D_\mu\Delta = \partial_\mu\Delta + i\sqrt{\frac{3}{5}}g_1 B_\mu\Delta + ig_2\left[W_\mu,\Delta\right] \tag{6}$$

and C is the charge conjugation matrix with respect to the Lorentz group. The counterterm parts of the Lagrangian which are needed in the paper are given in App. A. In addition, we consider an effective dimension 5 (D5) operator which generates neutrino masses because it appears as soon as the Higgs triplet or a right–handed neutrino decouples:

$$\mathscr{L}_\kappa = -\frac{1}{4}\kappa_{fg}\left(\overline{\ell_L^f} i\sigma_2\phi\right)\left(\ell_L^{g\,\mathrm{C}} i\sigma_2\phi\right) . \tag{7}$$

The most general neutrino mass matrix is given by the following formula

$$m_\nu = -\frac{v^2}{4}\left(\kappa + 2Y_\nu^T M^{-1}Y_\nu - 2\frac{Y_\Delta\Lambda_6}{M_\Delta^2}\right) , \tag{8}$$

where κ includes additional contributions to the dimension 5 operator, like from gravitational effects [62]. Thus the β-function of the neutrino mass is given by the sum of the β-functions for the contribution from the right-handed neutrinos and the contribution from the Higgs triplet.

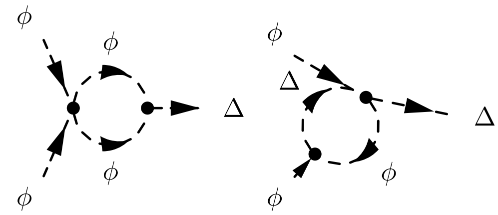


FIG. 1: Feynman diagrams which are not considered in [61].

The right–handed neutrinos and the Higgs triplet decouple step by step at their respective mass scale and the effective theories have to be matched against each other. The decoupling of the right–handed neutrinos only contributes to the effective D5 operator. The decoupling of the Higgs triplet also gives a contribution to the SM model Higgs self–coupling because there is a coupling between the SM Higgs doublet and the Higgs triplet given in Eq.(5c). Hence, the matching conditions of the right-handed neutrinos are

$$\kappa^{\mathrm{EFT}} = \kappa + 2Y_\nu^T M^{-1}Y_\nu \tag{9}$$

and the decoupling of the Higgs triplet leads to

$$\kappa^{\mathrm{EFT}} = \kappa - 2\frac{Y_\Delta\Lambda_6}{M_\Delta^2} \tag{10a}$$

$$\lambda^{\mathrm{EFT}} = \lambda + 2\frac{|\Lambda_6|^2}{M_\Delta^2} . \tag{10b}$$

In the following two sections, we present the calculation of the renormalization group equations. The calculation has been carefully double checked and compared to the results of Chao and Zhang [61].

III. WAVE FUNCTION RENORMALIZATION AND COUNTERTERMS

In order to obtain the wave function renormalization factor, all self–energy diagrams of the involved particles have to be calculated to one loop order. We use dimensional regularization together with the $\overline{MS}$ scheme, because gauge invariance is generically preserved in this scheme. The differences to the formulae in [61] are underlined. Singly underlined terms differ in the prefactor and wavy underlined terms are not present in [61]. Thus they probably stem from diagrams not taken into account in [61]. For example, the wavy underlined terms in the counterterm of Λ_6 are due to the Feynman diagrams shown in Fig.1. Our Feynman rules and calculation can be downloaded from `http://www.mpi-hd.mpg.de/~mschmidt/rgeTriplet/`. As we use a different definition for the couplings in the Lagrangian as in [61], the translation is presented in Tab. I. The wave function renormalization factors are

our work	Y_Δ	m	M_Δ	λ	Λ_1	Λ_2	Λ_4	Λ_5	Λ_6
Chao and Zhang [61]	$\frac{1}{\sqrt{2}}Y_\xi$	m_ϕ	M_ξ	λ	$\frac{1}{2}\lambda_\xi$	λ_C	λ_ϕ	$-\frac{1}{2}\lambda_T$	$-\frac{1}{\sqrt{2}}M_\xi\lambda_H$

TABLE I: Translation table for all relevant parameters

given by

$$\delta Z_\Delta = \frac{1}{16\pi^2\epsilon}\left[\frac{6}{5}\left(3-\xi_1\right)g_1^2 + 4\left(3-\xi_2\right)g_2^2 - 2\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\right] \tag{11a}$$

$$\delta Z_\phi = -\frac{1}{16\pi^2\epsilon}\left[2T - \frac{3}{10}\left(3-\xi_1\right)g_1^2 - \frac{3}{2}\left(3-\xi_2\right)g_2^2\right] \tag{11b}$$

$$\begin{aligned}\delta Z_{\ell_L} = &-\frac{1}{16\pi^2\epsilon}\left[Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3Y_\Delta^\dagger Y_\Delta\right]\\ &-\frac{1}{16\pi^2\epsilon}\left[\frac{3}{10}\xi_1 g_1^2 + \frac{3}{2}\xi_2 g_2^2\right]\ ,\end{aligned} \tag{11c}$$

where we have defined

$$T = \mathrm{tr}\left(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\ . \tag{12}$$

The Yukawa coupling vertex $\ell\Delta\ell$ can be renormalized multiplicatively

$$\delta Z_{Y_\Delta} = \frac{1}{32\pi^2\epsilon}\left[\frac{9}{5}\left(1-\xi_1\right)g_1^2 + \left(3-7\xi_2\right)g_2^2\right] \tag{13}$$

the parameters in the Higgs potential, however, have to be renormalized additively

$$\delta m^2 = \frac{1}{16\pi^2\epsilon}\left[\left(3\lambda - \frac{3}{10}\xi_1 g_1^2 - \frac{3}{2}\xi_2 g_2^2\right)m^2 - 4\,\mathrm{tr}\left(Y_\nu^\dagger M_\Delta^2 Y_\nu\right) + 6\Lambda_4 M_\Delta^2 + 6|\Lambda_6|^2\right] \tag{14a}$$

$$\delta M_\Delta^2 = \frac{1}{16\pi^2\epsilon}\left[\left(8\Lambda_1 + 2\Lambda_2 - \frac{6}{5}\xi_1 g_1^2 - 4\xi_2 g_2^2\right)M_\Delta^2 + 4\Lambda_4 m^2 + 2|\Lambda_6|^2\right] \tag{14b}$$

$$\begin{aligned}\delta\lambda = &\frac{1}{16\pi^2\epsilon}\Bigg[6\lambda^2 - \frac{1}{2}\lambda\left(\frac{3}{5}g_1^2 + 3g_2^2\right) + 3g_2^4 + \frac{3}{2}\left(\frac{3}{5}g_1^2 + g_2^2\right)^2\\ &-8\,\mathrm{tr}\left[Y_e^\dagger Y_e Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u Y_u^\dagger Y_u + 3Y_d^\dagger Y_d Y_d^\dagger Y_d\right] + 12\Lambda_4^2 + 8\Lambda_5^2\Bigg]\end{aligned} \tag{14c}$$

$$\begin{aligned}\delta\Lambda_1 = &\frac{1}{16\pi^2\epsilon}\Bigg[\left(-\frac{12}{5}\xi_1 g_1^2 - 8g_2^2\xi_2\right)\Lambda_1 + \frac{9}{25}12g_1^4\underline{+18g_2^4} + \frac{72}{5}g_1^2g_2^2\underline{+14\Lambda_1^2} + 2\Lambda_2^2 + 4\Lambda_1\Lambda_2 + 4\left(\Lambda_4^2+\Lambda_5^2\right)\\ &\underline{-8\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\Delta^\dagger Y_\Delta\right)}\Bigg]\end{aligned} \tag{14d}$$

$$\delta\Lambda_2 = \frac{1}{16\pi^2\epsilon}\left[\left(-\frac{12}{5}\xi_1 g_1^2 - 8g_2^2\xi_2\right)\Lambda_2\underline{+12g_2^4} - \frac{144}{5}g_1^2g_2^2 + 3\Lambda_2^2 + 12\Lambda_1\Lambda_2 - 8\Lambda_5^2\underline{+8\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\Delta^\dagger Y_\Delta\right)}\right] \tag{14e}$$

$$\delta\Lambda_4 = \frac{1}{16\pi^2\epsilon}\left[\left(-\frac{3}{2}\xi_1 g_1^2 - \frac{11}{2}\xi_2 g_2^2\right)\Lambda_4 + \frac{27}{25}g_1^4\underline{+6g_2^4} + \left(8\Lambda_1 + 2\Lambda_2 + 3\lambda + 4\Lambda_4\right)\Lambda_4 + 8\Lambda_5^2 - 4\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\nu^\dagger Y_\nu\right)\right] \tag{14f}$$

$$\delta\Lambda_5 = \frac{1}{16\pi^2\epsilon}\left[\left(-\frac{3}{2}\xi_1 g_1^2 - \frac{11}{2}\xi_2 g_2^2\right)\Lambda_5 - \frac{18}{5}g_1^2g_2^2 + \left(2\Lambda_1 - 2\Lambda_2 + \lambda + 8\Lambda_4\right)\Lambda_5 + 4\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\nu^\dagger Y_\nu\right)\right] \tag{14g}$$

$$\delta\Lambda_6 = -\frac{1}{16\pi^2\epsilon}\left[4\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\nu^T M Y_\nu\right) + \left(\frac{9}{10}g_1^2\xi_1 + \frac{7}{2}g_2^2\xi_2\underline{-\lambda + 4\Lambda_4 - 8\Lambda_5}\right)\Lambda_6\right]\ . \tag{14h}$$

IV. RENORMALIZATION GROUP EQUATIONS

Using the vertex corrections and the wave function renormalization factors, we can deduce the β–functions via the formula given in [63]. As the wave function renormalization constant for the left-handed lepton doublets has an additional term with respect to the SM extended by right-handed neutrinos, all vertices receive an additional contribution per left-handed lepton attached to the vertex which is given by

$$\frac{3}{2}\frac{1}{16\pi^2}Y_\Delta^\dagger Y_\Delta \tag{15}$$

multiplied by the matrix characterizing the vertex from the left or from the right, respectively. In particular, the

β-functions of the lepton Yukawa couplings [64, 65, 66] and the Yukawa coupling Y_Δ become

$$16\pi^2 \dot{Y}_\nu = Y_\nu \left[\frac{3}{2}Y_\nu^\dagger Y_\nu - \frac{3}{2}Y_e^\dagger Y_e + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right] + Y_\nu \left[T - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2\right] \tag{16a}$$

$$16\pi^2 \dot{Y}_e = Y_e \left[\frac{3}{2}Y_e^\dagger Y_e - \frac{3}{2}Y_\nu^\dagger Y_\nu + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right] + Y_e \left[T - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right] \tag{16b}$$

$$16\pi^2 \dot{Y}_\Delta = \left[\frac{1}{2}Y_\nu^\dagger Y_\nu + \frac{1}{2}Y_e^\dagger Y_e + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right]^T Y_\Delta + Y_\Delta \left[\frac{1}{2}Y_\nu^\dagger Y_\nu + \frac{1}{2}Y_e^\dagger Y_e + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right] + \left[-\frac{3}{2}\left(\frac{3}{5}g_1^2 + 3g_2^2\right) + \mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\right] Y_\Delta\ . \tag{16c}$$

The renormalization of Λ_6 is described by

$$16\pi^2 \dot{\Lambda}_6 = \Big[\underset{\sim\sim\sim}{\lambda - 4\Lambda_4 + 8\Lambda_5} - \frac{27}{10}g_1^2 - \frac{21}{2}g_2^2 + 2T + \mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Big]\Lambda_6 - 2\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\nu^T M Y_\nu\right) \tag{17}$$

and the anomalous dimensions of the Higgs triplet mass term is given by

$$16\pi^2 \gamma_{M_\Delta} = \frac{9}{5}g_1^2 + 6g_2^2 - 4\Lambda_1 - \Lambda_2 - \mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right) + \left(-2\Lambda_4 m^2 - \frac{1}{2}|\Lambda_6|^2\right) M_\Delta^{-2}\ . \tag{18}$$

The β-function of the effective neutrino mass operator κ function changes to

$$16\pi^2 \dot{\kappa} = \left[\frac{1}{2}Y_\nu^\dagger Y_\nu - \frac{3}{2}Y_e^\dagger Y_e + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right]^T \kappa + \kappa \left[\frac{1}{2}Y_\nu^\dagger Y_\nu - \frac{3}{2}Y_e^\dagger Y_e + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right] + \left[2T - 3g_2^2 + \lambda\right]\kappa\ . \tag{19}$$

The RG equation for the type I contribution to the neutrino mass is only changed by the additional term to the β-function of the neutrino Yukawa couplings due to the Higgs triplet. The remaining RG equations are presented in App. B. They either do not receive additional contributions or do not directly influence the neutrino mass matrix. The main difference in the RG equations compared to the results in [61] arise from contributions of the additional diagrams contributing to Λ_6. As it can been seen later, they have an impact on the evolution of neutrino masses, but the evolution of mixing angles and phases remains unchanged. In summary, the running of the effective neutrino mass matrix m_ν above and between the seesaw scales is given by the running of the three different contributions to the neutrino mass matrix,

$$\begin{aligned} m_\nu^{(1)} &= -\frac{v^2}{4}\kappa, \\ m_\nu^{(2)} &= -\frac{v^2}{2}Y_\nu^T M^{-1} Y_\nu, \\ m_\nu^{(3)} &= \frac{v^2}{2}\Lambda_6 M_\Delta^{-2} Y_\Delta\ . \end{aligned} \tag{20}$$

The 1-loop β-functions for m_ν in the various effective theories can be summarized as

$$\begin{aligned} 16\pi^2 \frac{\mathrm{d}m_\nu^{(i)}}{\mathrm{d}t} = &\left[C_e Y_e^\dagger Y_e + C_\nu Y_\nu^\dagger Y_\nu + C_\Delta Y_\Delta^\dagger Y_\Delta\right]^T m_\nu^{(i)} \\ &+ m_\nu^{(i)}\left[C_e Y_e^\dagger Y_e + C_\nu Y_\nu^\dagger Y_\nu + C_\Delta Y_\Delta^\dagger Y_\Delta\right] \\ &+ \alpha\, m_\nu^{(i)}\ , \end{aligned} \tag{21}$$

where $m_\nu^{(i)}$ stands for any of the three contributions to the neutrino mass matrix, respectively. The coefficients $C_{e,\nu,\Delta}$ and α are listed in Tab. II. In the type-I+II seesaw scenario, large RG effects can be expected before the Higgs triplet is integrated out due to the different coefficients (C_e, C_ν, C_Δ) in analogy to the standard seesaw scenario where large RG corrections between the thresholds are induced by additional flavor–diagonal vertex corrections to the D5 operator.

V. HIGGS TRIPLET IN THE MSSM

In the MSSM, in addition to the Higgs triplet $\Delta \sim (\mathbf{3},\mathbf{1})$, a second Higgs triplet $\overline{\Delta} \sim (\mathbf{3},-\mathbf{1})$ with opposite hypercharge Y is needed to generate a D5 mass term for neutrinos. Furthermore, $\overline{\Delta}$ ensures that the model is anomaly-free. Note, however, that only one Higgs triplet couples to the left–handed leptons. The additional terms

model	$m_\nu^{(i)}$	C_e	C_ν	C_Δ	flavor-trivial term α
SM	κ	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$2T - 3g_2^2 + \lambda$
SM	$2\,Y_\nu^T M^{-1} Y_\nu$	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$2T - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2$
SM	$-2\,\Lambda_6 M_\Delta^{-2} Y_\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$T - 2\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right) - 3g_2^2 + \lambda - 8\Lambda_1 - 2\Lambda_2 - 4\Lambda_4 + 8\Lambda_5 -$ $\left(4\Lambda_4 m^2 + \lvert\Lambda_6\rvert^2\right) M_\Delta^{-2} - 4\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\nu^T M Y_\nu\right) \Lambda_6^{-1}$
MSSM	κ	1	3	$\frac{3}{2}$	$2\,\mathrm{tr}\left(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u\right) + 3\lvert\Lambda_u\rvert^2 - 2\left(\frac{3}{5}g_1^2 + 3g_2^2\right)$
MSSM	$2\,Y_\nu^T M^{-1} Y_\nu$	1	3	$\frac{3}{2}$	$2\,\mathrm{tr}\left(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u\right) + 3\lvert\Lambda_u\rvert^2 - 2\left(\frac{3}{5}g_1^2 + 3g_2^2\right)$
MSSM	$-2\,\Lambda_6 M_\Delta^{-2} Y_\Delta$	1	3	$\frac{3}{2}$	$2\,\mathrm{tr}\left(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u\right) + 3\lvert\Lambda_u\rvert^2 - 2\left(\frac{3}{5}g_1^2 + 3g_2^2\right)$

TABLE II: Coefficients of the β-functions of Eq. (21), which govern the running of the effective neutrino mass matrix in minimal type II seesaw models. In the MSSM, the coefficients coincide due to the non-renormalization theorem [67, 68] in supersymmetric theories.

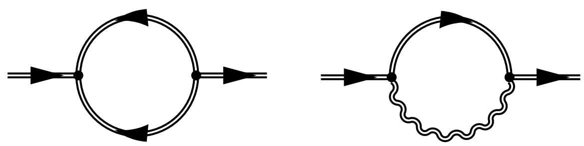


FIG. 2: Supergraphs which contribute to the wave function renormalization.

in the superpotential are given by

$$\begin{aligned} W_\Delta =& M_\Delta \,\mathrm{Tr}(\overline{\mathbb{\Delta}}\mathbb{\Delta}) + \frac{(Y_\Delta)_{fg}}{\sqrt{2}} \mathbb{l}^{fT} i\sigma_2 \mathbb{\Delta} \mathbb{l}^g \\ &+ \frac{\Lambda_u}{\sqrt{2}} \mathbb{h}^{(2)T} i\sigma_2 \overline{\mathbb{\Delta}} \mathbb{h}^{(2)} + \frac{\Lambda_d}{\sqrt{2}} \mathbb{h}^{(1)T} i\sigma_2 \mathbb{\Delta} \mathbb{h}^{(1)} , \end{aligned} \tag{22}$$

where $\mathbb{l}$ denotes the left-handed doublet and $\mathbb{h}^{(i)}$ denotes the Higgs doublets. We use the same notation as in [69].

The decoupling of the Higgs triplet generates an effective dimension 4 term κ^{EFT} in the superpotential, whereas the tree-level matching condition reads

$$\kappa^{\mathrm{EFT}} = \kappa - 2\frac{Y_\Delta \Lambda_u}{M_\Delta} . \tag{23}$$

The RG equations [80] can be obtained easily by using the supergraph technique as it is described in [69]. There are only two different types of supergraphs contributing to the wave function renormalization which are shown in Fig. 2. Here, we only show the RG equations which are relevant for the RG evolution of the neutrino mass matrix

$$16\pi^2 \dot{Y}_\Delta = Y_\Delta \left[Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + \frac{3}{2} Y_\Delta^\dagger Y_\Delta\right] + \left[Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + \frac{3}{2} Y_\Delta^\dagger Y_\Delta\right]^T Y_\Delta + Y_\Delta \left[\frac{1}{2}\lvert\Lambda_d\rvert^2 + \frac{1}{2}\,\mathrm{tr}(Y_\Delta^\dagger Y_\Delta) - \frac{9}{5}g_1^2 - 7g_2^2\right] \tag{24}$$

$$16\pi^2 \dot{\Lambda}_u = \Lambda_u \left[2\,\mathrm{tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) + \frac{7}{2}\lvert\Lambda_u\rvert^2 - \frac{9}{5}g_1^2 - 7g_2^2\right] \tag{25}$$

$$16\pi^2 \dot{M}_\Delta = M_\Delta \left[\frac{1}{2}\,\mathrm{tr}(Y_\Delta^\dagger Y_\Delta) + \frac{1}{2}\lvert\Lambda_u\rvert^2 + \frac{1}{2}\lvert\Lambda_d\rvert^2 - 4\left(\frac{3}{5}g_1^2 + 2g_2^2\right)\right] . \tag{26}$$

As it can be seen in the β-function of Y_Δ for example, the sign of $Y_\Delta^\dagger Y_\Delta$ in P equals the one in the SM which leads to the same sign in the RG equations of the angles in the limit of a strong hierarchy (See Sec. VI). The RG equation of the effective D5 operator κ is

$$16\pi^2 \dot{\kappa} = \kappa \left[Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + \frac{3}{2} Y_\Delta^\dagger Y_\Delta\right] + \left[Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + \frac{3}{2} Y_\Delta^\dagger Y_\Delta\right]^T \kappa + \kappa \left[2\,\mathrm{tr}(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u) + 3\lvert\Lambda_u\rvert^2 - 2\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\right] . \tag{27}$$

There is also an additional contribution to the RG equations which are relevant for the type I contribution to neutrino masses

$$16\pi^2\dot{Y}_\nu = Y_\nu\left[Y_e^\dagger Y_e + 3Y_\nu^\dagger Y_\nu + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right] + Y_\nu\left[\mathrm{tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) + \frac{3}{2}|\Lambda_u|^2 - \left(\frac{3}{5}g_1^2 + 3g_2^2\right)\right] \tag{28}$$

$$16\pi^2\dot{M} = M\left(2Y_\nu Y_\nu^\dagger\right)^T + \left(2Y_\nu Y_\nu^\dagger\right)M\ . \tag{29}$$

All remaining RG equations are presented in App. C.

The coefficients of the RG equations of the different contributions to the neutrino mass matrix are summarized in Tab. II. Note, that the coefficients C_e, C_ν, C_Δ and α are the same for all three mass contributions $m_\nu^{(i)}$ in the MSSM due to the non-renormalization theorem [67, 68].

VI. RG EQUATIONS OF MIXING PARAMETERS IN THE TYPE II SEESAW CASE

In order to understand the RG evolution of neutrino masses and leptonic mixing parameters in the presence of a Higgs triplet, we consider a type II model, where the neutrino mass is generated by a Higgs triplet only. The evolution of the mixing parameters in standard parameterization can be described by the formulae of [52] with suitable replacements for P, F, α and α_e:

$$\dot{m}_\nu = P^T m_\nu + m_\nu P + \alpha\, m_\nu \tag{30a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}Y_e^\dagger Y_e = F^\dagger Y_e^\dagger Y_e + Y_e^\dagger Y_e F + \alpha_e Y_e^\dagger Y_e \tag{30b}$$

Here, we can express P and F in terms of physical parameters.

$$P = C_e\,\mathrm{diag}(y_e^2, y_\mu^2, y_\tau^2) + C_\Delta U^*\,\mathrm{diag}(y_1^2, y_2^2, y_3^2)U^T \tag{31}$$

$$F = D_e\,\mathrm{diag}(y_e^2, y_\mu^2, y_\tau^2) + D_\Delta U^*\,\mathrm{diag}(y_1^2, y_2^2, y_3^2)U^T\ , \tag{32}$$

where U is the MNS matrix and $y_i = \frac{m_i}{v_\Delta}$. We use the so-called standard-parameterization [71]

$$\begin{aligned} U &= \mathrm{diag}(e^{\mathrm{i}\delta_e}, e^{\mathrm{i}\delta_\mu}, e^{\mathrm{i}\delta_\tau})V\,\mathrm{diag}(e^{-\mathrm{i}\varphi_1/2}, e^{-\mathrm{i}\varphi_2/2}, 1) \\ V &= R_{23}(\theta_{23})\Gamma_\delta^\dagger R_{13}(\theta_{13})\Gamma_\delta R_{12}(\theta_{12})\ , \end{aligned} \tag{33}$$

where R_{ij} is the matrix of rotation in the $i-j$ - plane and $\Gamma_\delta = \mathrm{diag}(e^{\mathrm{i}\delta/2}, 1, e^{-\mathrm{i}\delta/2})$. Note, that the Majorana phases drop out of the definition of P and F in flavor basis. Hence the RG equations of the angles and the Dirac CP phase are independent of the Majorana phases, as it can be seen below. We derive the RG equations by using the technique described in the appendix of [52] which is based on earlier works [53, 72, 73]. In the numerical examples which are shown in the figures, we do not include any finite threshold corrections, since we are considering 1 loop running and the finite threshold corrections are assumed to be of the order of 2 loop RG running. Therefore, the Higgs triplet is decoupled when its running mass equals the renormalization scale

$$\mu_{\mathrm{dec}} = M_\Delta(\mu_{\mathrm{dec}})\ . \tag{34}$$

In all examples, we set $M_\Delta(\Lambda_{\mathrm{GUT}}) = 10^{10}\,\mathrm{GeV}$. As we are only interested in showing the generic features of the RG evolution, we choose the Higgs self-couplings to be $\Lambda_{1,2,4,5} = 0.5$ for simplicity, since they only indirectly influence the RG evolution of the angles and the flavor-dependent part of the RG equations of the masses. In a realistic model, the parameters Λ_i have to satisfy certain relations to produce the desired vevs.

In the following, we present all formulae in the approximation $y_e \ll y_\mu \ll y_\tau$ and $\theta_{13} \ll 1$. The exact formulae can be downloaded from `http://www.mpi-hd.mpg.de/~mschmidt/rgeTriplet`.

A. Running of the masses

The main contributions to the RG equations of the masses

$$16\pi^2\frac{\dot{m}_1}{m_1} = \mathrm{Re}\,\alpha + 2C_\Delta\frac{m_1^2}{v_\Delta^2} + 2C_e y_\tau^2\sin^2\theta_{12}\sin^2\theta_{23} + \mathcal{O}(\theta_{13}) \tag{35a}$$

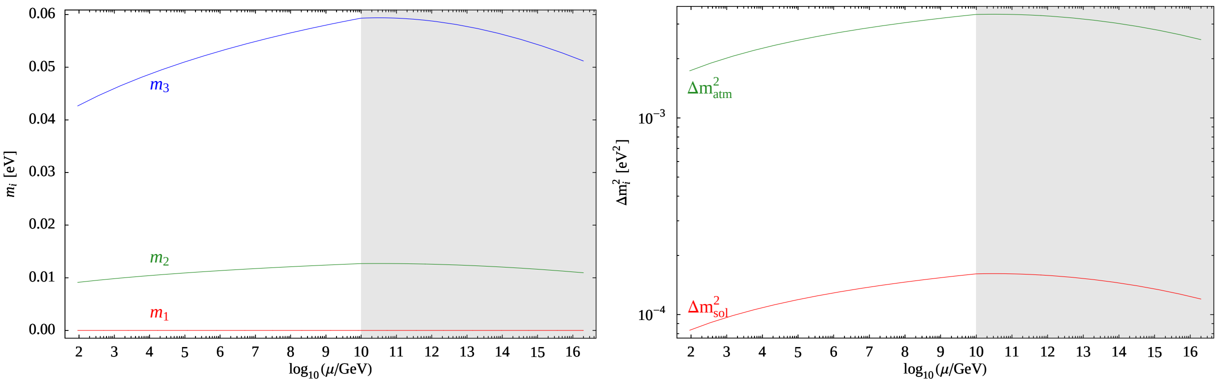
$$16\pi^2\frac{\dot{m}_2}{m_2} = \mathrm{Re}\,\alpha + 2C_\Delta\frac{m_2^2}{v_\Delta^2} + 2C_e y_\tau^2\cos^2\theta_{12}\sin^2\theta_{23} + \mathcal{O}(\theta_{13}) \tag{35b}$$

$$16\pi^2\frac{\dot{m}_3}{m_3} = \mathrm{Re}\,\alpha + 2C_\Delta\frac{m_3^2}{v_\Delta^2} + 2C_e y_\tau^2\cos^2\theta_{23} + \mathcal{O}(\theta_{13}) \tag{35c}$$

are the flavor–independent term $\mathrm{Re}\,\alpha$ and the flavor–dependent term $2\,C_\Delta\frac{m_i^2}{v_\Delta^2}$. As the smallness of neutrino masses is usually explained by a small vev of the Higgs triplet v_Δ, the singular values $y_i = \frac{m_i}{v_\Delta}$ of the Yukawa coupling Y_Δ can be of $\mathcal{O}(1)$. This in turn leads to sizable RG effects. Furthermore, the evolution of the mass squared difference is mainly given by

$$16\pi^2\frac{\dot{\Delta m^2_{ji}}}{\Delta m^2_{ji}} \approx 2\,\mathrm{Re}\,\alpha + 4C_\Delta\frac{m_j^2 + m_i^2}{v_\Delta^2} \tag{36}$$

in the SM and MSSM with small $\tan\beta$. There can be a cancellation of the RG effect depending on the parameters Λ_i in the Higgs potential and the sign of C_Δ, but



(a)Evolution of neutrino masses (b)Evolution of the mass squared differences

FIG. 3: As input values, we have chosen tribimaximal mixing at the GUT scale, $m_1 = 0\,\mathrm{eV}$, $\Delta m^2_{\mathrm{atm}} = 2.5\,10^{-3}\,\mathrm{eV}^2$, $\Delta m^2_{\mathrm{sol}} = 1.2\,10^{-4}\,\mathrm{eV}^2$, $M_\Delta = 10^{10}$ GeV and $\Lambda_6 = 2.5\,10^{-5} M_\Delta$, corresponding to $\langle\Delta\rangle = 0.15\,\mathrm{eV}$. As we are only interested in showing the generic feature of the RG evolution, we choose the Higgs self-couplings to be $\Lambda_{1,2,4,5} = 0.5$ for simplicity, since they only indirectly influence the RG evolution of the angles and the flavor-dependent part of the RG equations of the masses. The shadowed area indicates the region where the Higgs triplet is present. It is integrated out at the border between the shadowed and the white area.

generically the RG effect in the effective theory is large, as it can be seen in Fig. 3. This is just one possible example. The precise RG effect strongly depends on the parameters in the Higgs potential Λ_i. The charged lepton Yukawa couplings depend on the singular values of Yukawa coupling matrix Y_Δ in a flavor non–diagonal way:

$$16\pi^2 \frac{\dot{m}_e}{m_e} = \mathrm{Re}\,\alpha_e + D_\Delta \left(\frac{m_1^2}{v_\Delta^2} \cos^2\theta_{12} + \frac{m_2^2}{v_\Delta^2} \sin^2\theta_{12} \right) + \mathcal{O}(\theta_{13}) \tag{37a}$$

$$16\pi^2 \frac{\dot{m}_\mu}{m_\mu} = \mathrm{Re}\,\alpha_e + D_\Delta \left[\frac{m_3^2}{v_\Delta^2} \sin^2\theta_{23} + \left(\frac{m_2^2}{v_\Delta^2} \cos^2\theta_{12} + \frac{m_1^2}{v_\Delta^2} \sin^2\theta_{12} \right) \cos^2\theta_{23} \right] + \mathcal{O}(\theta_{13}) \tag{37b}$$

$$16\pi^2 \frac{\dot{m}_\tau}{m_\tau} = \mathrm{Re}\,\alpha_e + D_\Delta \left[\frac{m_3^2}{v_\Delta^2} \cos^2\theta_{23} + \left(\frac{m_2^2}{v_\Delta^2} \cos^2\theta_{12} + \frac{m_1^2}{v_\Delta^2} \sin^2\theta_{12} \right) \sin^2\theta_{23} \right] + D_e y_\tau^2 + \mathcal{O}(\theta_{13})\,. \tag{37c}$$

B. Running of the mixing angles

Chao and Zhang [61] have derived the formulae in the approximation $|Y_e| \ll |Y_\Delta|$ which captures the dominant effects as long as there is a strong hierarchy. Here, we calculate the renormalization group equations exactly [81] and present the equations for the mixing angles in the approximation of vanishing y_e, y_μ and θ_{13}:

$$16\pi^2 \dot{\theta}_{12} = -\frac{1}{2}\left[D_\Delta \frac{\Delta m^2_{21}}{v_\Delta^2} + C_e y_\tau^2 \frac{(m_2+m_1)^2}{\Delta m^2_{21}} \sin\theta_{23} \right] \sin 2\theta_{12} + \mathcal{O}(\theta_{13}) \tag{38a}$$

$$16\pi^2 \dot{\theta}_{13} = -\frac{C_e}{2} y_\tau^2 \frac{(m_2-m_1)m_3}{(m_3-m_1)(m_3-m_2)} \cos\delta \sin 2\theta_{12} \sin 2\theta_{23} + \mathcal{O}(\theta_{13}) \tag{38b}$$

$$16\pi^2 \dot{\theta}_{23} = -\frac{1}{2}\left[D_\Delta \left(\frac{m_3^2}{v_\Delta^2} - \frac{m_1^2}{v_\Delta^2} \sin^2\theta_{12} - \frac{m_2^2}{v_\Delta^2} \cos^2\theta_{12} \right) + C_e y_\tau^2 \frac{m_3^2 - m_1 m_2 + (m_2-m_1)\, m_3 \cos 2\theta_{12}}{(m_3-m_2)\,(m_3-m_1)} \right] \sin 2\theta_{23} + \mathcal{O}(\theta_{13})\,, \tag{38c}$$

where C_e and D_Δ are defined in Eqs. (31,32). The two contributions to the running from charged leptons and neutrinos can be of the same order of magnitude and

it strongly depends on the hierarchy of neutrino masses which of the two contributions is dominant. The contribution coming from the neutrino mass matrix ($\propto C_e$) shows almost the same features as in the effective theory:

- there is an enhancement factor which is proportional to $\frac{m_0^2}{\Delta m_{ji}^2}$, where m_0 denotes the mass scale of neutrinos;
- the running strongly depends on $\tan\beta$ due to the charged lepton Yukawa couplings;
- vanishing mixing is a fixed point.

In contrast to the effective theory, however, there is no dependence on Majorana phases. This still holds for the exact equations. The RG evolution of the mixing angles is only influenced by the Dirac CP phase. On the other hand, the contribution from the charged leptons shows a completely different dependence on the Yukawa couplings. It is basically proportional to the corresponding mass squared difference divided over the vev of the Higgs triplet squared. Hence, there is no large enhancement factor and no dependence on $\tan\beta$ in the SUSY case. Thus the overall size of the RG effect mainly depends on the vev of the Higgs triplet.

$$\dot{\theta}_{ij} \sim \frac{\Delta m_{ji}^2}{v_\Delta^2} \sin 2\theta_{ij} \tag{39}$$

This gives a good estimate for the running in the strongly hierarchical case. The sign of the RG effect is determined by the sign of the mass squared difference and the factor D_Δ in front of the factor $Y_\Delta^\dagger Y_\Delta$ in P. As D_Δ is positive in the SM and MSSM, θ_{23} is evolving to larger values coming from the high renormalization scale for a normal hierarchy. Furthermore, the β–function is approximately proportional to $\sin 2\theta_{ij}$ which implies that a vanishing angle remains small. Taking into account these generic features, the RG effect from the charged leptons is largest on θ_{23} due to the combination of a large mass squared difference and a large angle. As it can be seen from the equations, zero mixing is a fixed point. This is also obvious from the RG equation in matrix form: in this configuration, P and F will be diagonal, if Y_e and Y_ν are diagonal. In Fig.4, we have plotted the evolution of mixing angles in the SM for a strongly hierarchical spectrum in order to suppress the effect coming from the effective D5 operator. The gross features of the running can be immediately seen: the only sizable effect is on θ_{23} due to the large angle and mass squared difference. As it can be seen from the plot, the RG effect can be estimated by a leading log approximation to

$$\Delta\theta_{ij} \approx -\frac{D_\Delta}{2}\frac{\Delta m_{ji}^2}{v_\Delta^2} \sin 2\theta_{ij} \ln\frac{\Lambda}{M_\Delta} \ . \tag{40}$$

The contribution to θ_{13} coming from the charged leptons vanishes in our approximation. For non–vanishing θ_{13}, it is given by

$$-\frac{D_\Delta}{2}\left(\frac{m_3^2}{v_\Delta^2} - \frac{m_1^2}{v_\Delta^2}\cos^2\theta_{12} - \frac{m_2^2}{v_\Delta^2}\sin^2\theta_{12}\right)\sin 2\theta_{13} \tag{41}$$

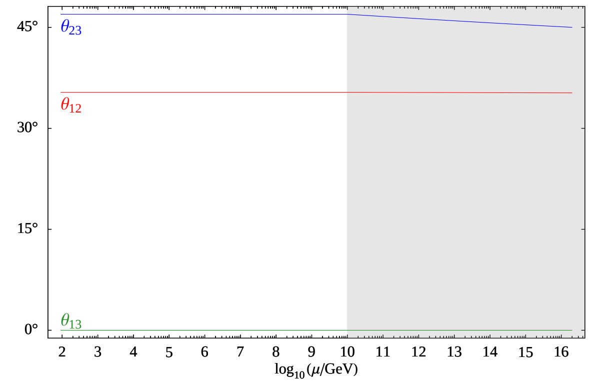


FIG. 4: Plot showing the evolution of the leptonic mixing angles in the SM. As input values, we have chosen tribimaximal mixing at the GUT scale, $m_1 = 0\,\text{eV}$, $\Delta m^2_{\text{atm}} = 2.5\,10^{-3}\,\text{eV}^2$, $\Delta m^2_{\text{sol}} = 1.2\,10^{-4}\,\text{eV}^2$, $M_\Delta = 10^{10}\,\text{GeV}$ and $\Lambda_6 = 2.5\,10^{-5} M_\Delta$, corresponding to $\langle\Delta\rangle = 0.15\,\text{eV}$. As we are only interested in showing the generic feature of the RG evolution, we choose the Higgs self-couplings to be $\Lambda_{1,2,4,5} = 0.5$ for simplicity (In a realistic model, the parameters Λ_i have to satisfy certain relations to produce the desired vevs.), since they only indirectly influence the RG evolution of the angles and the flavor-dependent part of the RG equations of the masses. In the MSSM with small $\tan\beta$, the plot looks similar due to C_Δ being the same in both theories. The shadowed area indicates the region where the Higgs triplet is present. It is integrated out at the border between the shadowed and the white area.

Let us comment on the configuration $\theta_{13} = m_3 = 0$, which is stable under the RG in the effective theory. Vanishing mass eigenvalues remain zero, as it can be seen from Eq. (35c), but θ_{13} receives corrections

$$\begin{aligned}16\pi^2\dot{\theta}_{13} =& \frac{C_e}{2}\frac{\Delta m_{21}^2}{v_\Delta^2}\frac{y_e^2\left(y_\tau^2 - y_\mu^2\right)}{\left(y_\tau^2 - y_e^2\right)\left(y_\mu^2 - y_e^2\right)}\cos\delta\sin 2\theta_{12}\\ &\times \sin 2\theta_{23} + \mathcal{O}(\theta_{13},\, y_3)\end{aligned} \tag{42}$$

Thus $\theta_{13} = m_3 = 0$ is not stable under the RG. However, the effect is negligible, because $\left(\frac{y_e}{y_\mu}\right)^2 \frac{\Delta m^2_{\text{sol}}}{\langle\Delta\rangle^2}$ is very small and $m_3 = 0$ is stable.

C. Running of the phases

The RG evolution of the phases is rather small:

$$16\pi^2\dot{\delta} = \frac{C_e}{2}\frac{(m_2-m_1)m_3}{(m_3-m_1)(m_3-m_2)}y_\tau^2\sin\delta\sin 2\theta_{12}\sin 2\theta_{23}\;\theta_{13}^{-1}+\mathcal{O}(\theta_{13}) \tag{43a}$$

$$16\pi^2\dot{\varphi}_1 = -2\left[2C_e y_\tau^2\frac{(m_1^2+m_3^2)m_2\sin^2\theta_{23}-((m_1^2+m_2^2)\sin^2\theta_{12}-m_1m_2(\cos 2\theta_{12}-\cos 2\theta_{23}))m_3}{(m_3-m_1)(m_3-m_2)(m_2-m_1)}\cot\theta_{12} + D_\Delta\frac{\Delta m_{21}^2}{v_\Delta^2}\sin 2\theta_{12}\right]\cot\theta_{23}\sin\delta\;\theta_{13}+\mathcal{O}(\theta_{13}^2) \tag{43b}$$

$$16\pi^2\dot{\varphi}_2 = -2\left[2C_e y_\tau^2\frac{-(m_1^2+m_2^2)m_3\cos^2\theta_{12}+m_1((m_2^2+m_3^2)\sin^2\theta_{23}+m_2m_3(\cos 2\theta_{12}+\cos 2\theta_{23}))}{(m_3-m_1)(m_3-m_2)(m_2-m_1)}\tan\theta_{12} + D_\Delta\frac{\Delta m_{21}^2}{v_\Delta^2}\sin 2\theta_{12}\right]\cot\theta_{23}\sin\delta\;\theta_{13}+\mathcal{O}(\theta_{13}^2)\;, \tag{43c}$$

because the leading order of the Majorana phases is of order θ_{13}. Only the Dirac CP phase δ involves a term which is inversely proportional to θ_{13}. Thus, there is a sizable effect for small θ_{13}. For vanishing θ_{13}, δ has to vanish (for realistic values of θ_{12} and θ_{23}) in order to ensure analyticity of $\delta(t)$. The RG equation of the Dirac CP phase δ does also not depend on the Majorana phases φ_i.

VII. RG EVOLUTION IN THE FULL TYPE II SEESAW CASE

In the full type II case, it is not possible to express the RG equations in terms of mixing parameters. Therefore one has to resort to numerical calculations. For this purpose, we have extended the Mathematica package REAP, which is available on the web page `http://www.ph.tum.de/~rge`, to include a left–handed triplet.

To illustrate the largeness of RG effects in the full type II seesaw scenario, we show an example, where bimaximal mixing at high energy evolves to the LMA solution at low energy. In previous works [30, 31, 32, 52], this evolution was due to an inverted hierarchy in the neutrino Yukawa couplings Y_ν or large imaginary off-diagonal entries. Here, the relevant matrix $Y_\nu^\dagger Y_\nu$ is real and has a normal hierarchy. In addition, the singular values of the Yukawa coupling matrix Y_Δ are small ($\mathcal{O}(10^{-5})$). In spite of the small couplings, there is a sizable effect on θ_{12} which can be seen in Fig. 5. It is due to the different RG equations of the contributions to the neutrino mass matrix.

In our example, we have chosen Λ_6 to be relatively large $\Lambda_6=\mathcal{O}(10^9)\,\mathrm{GeV}$, because it receives corrections of the order of $M_3\,(Y_\nu)_{33}^2\,(Y_\Delta)_{33}$. The evolution of the mixing angles θ_{12} and θ_{23} is highly non–linear above the threshold of the Higgs triplet. Hence, a leading log approximation is not possible. In the MSSM, the equations for the mixing angles presented in [52] are valid at each renormalization scale μ. Hence, θ_{12} is increasing, as long as there there are no imaginary off-diagonal entries and there is a normal hierarchy in the neutrino Yukawa couplings.

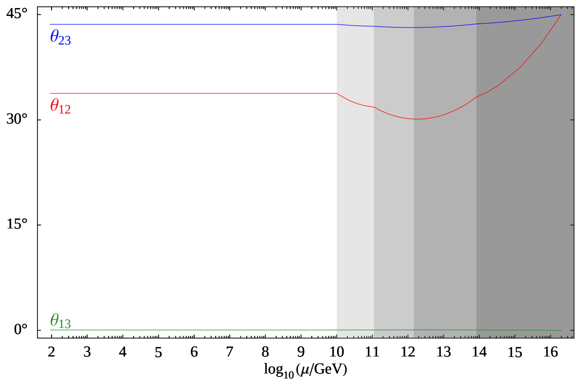
VIII. CONCLUSIONS

We calculated the RG equations in the type II seesaw case and found differences to the ones calculated by Chao and Zhang [61] in the RG equations of the parameters of the Higgs potential.

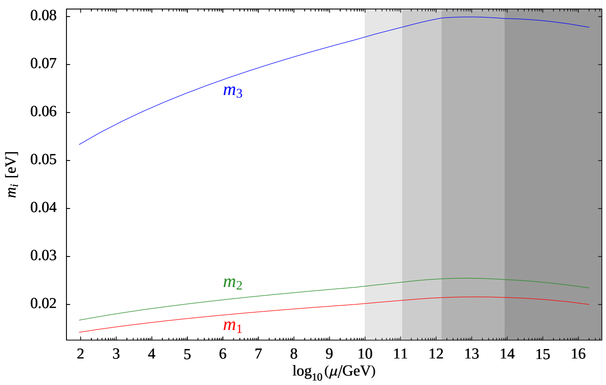
In the SM, the matrix P describing the off–diagonal contributions to the RG evolution of the neutrino mass matrix is different for the contribution coming from the Higgs triplet compared to the one for the effective D5 operator. Hence, there can be large RG effects due to the different RG equations of the different contributions to the neutrino mass matrix.

Furthermore, we derived the exact RG equations in terms of the mixing parameters. The equations have a different structure compared to the ones in the standard seesaw case as well as in the effective theory case. One feature is that the RG equations of the mixing angles are independent of the Majorana phases. Therefore, there is no damping of the RG effect due to Majorana phases. The main difference to the running in the standard seesaw scenario is the proportionality of the β-function of the mixing angles to the mass squared difference in contrast to the inverse proportionality in the case of a hierarchical spectrum. Hence, there is no enhancement factor and the RG effect is small as long as Y_Δ is small. Furthermore, the RG effect of the mixing angles θ_{ij} is proportional to $\sin 2\theta_{ij}$ which leads together with the proportionality to the mass squared difference to a large effect on θ_{23} compared to the effect on θ_{12} due to $\Delta m^2_{\mathrm{sol}} \ll \Delta m^2_{\mathrm{atm}}$ and compared to θ_{13} due to $\sin 2\theta_{13} \ll \sin 2\theta_{23}$.

The RG equations in the full case can only be studied numerically. The interplay of the contributions from



(a)Angles



(b)Masses

FIG. 5: In the full type II seesaw case, there is a complicated interplay between the two contributions to the neutrino mass matrix. Here, we just plot an example for the following initial values at the GUT scale: $M_\Delta = 10^{10}\,\text{GeV}$, $\Lambda_6 = 4.56\ 10^9\,\text{GeV}$, $m_1 = 0.02\,\text{eV}$, $\Delta m^2_{\text{sol}} = 1.5\ 10^{-4}\,\text{eV}^2$, $\Delta m^2_{\text{atm}} = 5.5\ 10^{-3}\,\text{eV}^2$, $\theta_{12} = \theta_{23} = \frac{\pi}{4}$, $\theta_{13} = 0$, $\delta = \varphi_1 = \varphi_2 = 0$, $Y_\nu = 0.37\,\text{diag}\left(10^{-2},\ 10^{-1},\ 1\right)$, where Y_Δ is chosen diagonal $Y_\Delta = \text{diag}\left(1.3\ 10^{-5},\ 1.5\ 10^{-5},\ 5.1\ 10^{-5}\right)$ and M is chosen appropriately to produce bimaximal mixing. The differently shaded areas indicate the different energy ranges of the various effective field theories. At each border, a particle, either a right-handed neutrino or the Higgs triplet, is integrated out.

right–handed neutrinos and the Higgs triplet can lead to large RG effects even in the SM. Hence, it is necessary to consider RG effects in model building to make predictions which can be compared to the experimental data.

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APPENDIX A: COUNTERTERM LAGRANGIAN

The relevant wave function renormalization factors are defined in the usual way by

$$\left(\ell_L\right)_B = Z_{\ell_L}^{\frac{1}{2}} \ell_L \tag{A1}$$

$$\Delta_B = Z_\Delta^{\frac{1}{2}} \Delta \tag{A2}$$

$$\phi_B = Z_\phi^{\frac{1}{2}} \phi\ . \tag{A3}$$

The Yukawa couplings are renormalized multiplicatively

$$\left(\left(Z_{\ell_L}^T\right)^{\frac{1}{2}} \left(Y_\Delta\right)_B Z_{\ell_L}^{\frac{1}{2}} Z_\Delta^{\frac{1}{2}}\right)_{fg} = \mu^{\frac{\epsilon}{2}} \left(Y_\Delta Z_{Y_\Delta}\right)_{fg} \tag{A4}$$

$$\left(Z_{\nu_R}^{\frac{1}{2}} \left(Y_\nu\right)_B Z_\phi^{\frac{1}{2}} Z_{\ell_L}^{\frac{1}{2}}\right)_{fg} = \mu^{\frac{\epsilon}{2}} \left(Y_\nu Z_\nu\right)_{fg}\ . \tag{A5}$$

The parameters in the Higgs potential have to be renormalized additively

$$Z_\phi m_B^2 = m^2 + \delta m^2 \tag{A6a}$$

$$Z_\Delta \left(M_\Delta^2\right)_B = M_\Delta^2 + \delta M_\Delta^2 \tag{A6b}$$

$$Z_\phi^2 \lambda_B = \mu^\epsilon Z_\lambda \lambda \tag{A6c}$$

$$Z_\Delta^2 \left(\Lambda_{1,2}\right)_B = \mu^\epsilon \left(\Lambda_{1,2} + \delta\Lambda_{1,2}\right) \tag{A6d}$$

$$Z_\Delta Z_\phi \left(\Lambda_{4,5}\right)_B = \mu^\epsilon \left(\Lambda_{4,5} + \delta\Lambda_{4,5}\right) \tag{A6e}$$

$$\left(Z_\Delta^\dagger\right)^{\frac{1}{2}} \left(Z_\phi^T\right)^{\frac{1}{2}} \left(\Lambda_6\right)_B Z_\phi^{\frac{1}{2}} = \mu^{\frac{\epsilon}{2}} \left(\Lambda_6 + \delta\Lambda_6\right)\ . \tag{A6f}$$

The insertion of the definitions of the renormalized quantities into the bare Lagrangian yields the counterterm part of the Lagrangian which is needed to cancel the divergences. Thus the counterterm Lagrangian

$$\mathcal{L}_{\text{ct}} = \mathcal{C}_{\nu_R} + \mathcal{C}_\Delta + \mathcal{C}_{V(\Delta)} \tag{A7}$$

is given by

$$\mathcal{C}_{\nu_R} = \frac{1}{2}\overline{\nu_R^g}\left(i\partial\!\!\!/\right)\left[\left(\delta Z_{\nu_R}\right)_{gf} \mathrm{P_L} - \frac{1}{2}\overline{\nu_R}^g \left(\delta Z_M M\right)_{gf} \nu_R^f - \left(Y_\nu \delta Z_{Y_\nu}\right)_{gf} \overline{\nu_R}^g \tilde{\phi}^\dagger \ell_L^f + \left(\delta Z_{\nu_R}\right)_{gf} \mathrm{P_R}\right] \nu_R^f + \text{h.c.} \tag{A8a}$$

$$
\begin{aligned}
\mathcal{C}_\Delta =& \delta Z_\Delta \operatorname{tr}\left(D_\mu \Delta\right)^\dagger \left(D^\mu \Delta\right) + \left[i \operatorname{tr}\left(D_\mu\Delta\right)^\dagger \left(\delta Z_{g_1} g_1 B^\mu \Delta + \delta Z_{g_2} g_2 \frac{\sigma_i}{2}\left[W^{i\mu},\Delta\right]\right) + \text{h.c.}\right] \\
& - \frac{Y_\Delta}{\sqrt{2}} \delta Y_\Delta \overline{\ell_L}^{\mathrm{C}} \left(i\sigma_2\right) \Delta \ell_L - \mathcal{C}_{V(\Delta)} \qquad \text{(A8b)}\\
\mathcal{C}_{V(\Delta)} =& \delta M_\Delta^2 \operatorname{tr}\Delta^\dagger\Delta + \frac{\delta\Lambda_1}{2}\left(\operatorname{tr}\Delta^\dagger\Delta\right)^2 + \frac{\delta\Lambda_2}{2}\left[\left(\operatorname{tr}\Delta^\dagger\Delta\right)^2 - \operatorname{tr}\left(\Delta^\dagger\Delta\Delta^\dagger\Delta\right)\right] \\
& + \delta\Lambda_4 \phi^\dagger\phi \operatorname{tr}\Delta^\dagger\Delta + \delta\Lambda_5 \phi^\dagger\left[\Delta^\dagger,\Delta\right]\phi + \frac{\delta\Lambda_6}{\sqrt{2}}\phi^T\left(i\sigma_2\right)\Delta^\dagger\phi\ . \qquad \text{(A8c)}
\end{aligned}
$$

APPENDIX B: RG EQUATIONS IN THE SM

The remaining RG equations of Yukawa coupling matrices

$$
\begin{aligned}
16\pi^2 \dot{Y}_d =& Y_d\left[\frac{3}{2}Y_d^\dagger Y_d - \frac{3}{2}Y_u^\dagger Y_u\right] + Y_d\left[T - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right] \qquad \text{(B1)}\\
16\pi^2 \dot{Y}_u =& Y_u\left[\frac{3}{2}Y_u^\dagger Y_u - \frac{3}{2}\,Y_d^\dagger Y_d\right] + Y_u\left[T - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right], \qquad \text{(B2)}
\end{aligned}
$$

where $T = \operatorname{Tr}\left[Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right]$ and the right-handed neutrino mass matrix

$$
16\pi^2 \dot{M} = \left(Y_\nu Y_\nu^\dagger\right) M + M\left(Y_\nu Y_\nu^\dagger\right)^T\ . \qquad \text{(B3)}
$$

are listed for completeness. They are taken from [52]. The anomalous dimensions of the Higgs doublet mass is given by

$$
16\pi^2\gamma_m = \frac{9}{20}g_1^2 + \frac{9}{4}g_2^2 - \frac{3}{2}\lambda - T + \frac{2}{m^2}\operatorname{tr}\left(Y_\nu^\dagger M^2 Y_\nu\right) - 3\Lambda_4\frac{M_\Delta^2}{m^2} - 3\frac{|\Lambda_6|^2}{m^2}\ . \qquad \text{(B4)}
$$

and the renormalization of the remaining couplings in the Higgs potential are described by

$$
\begin{aligned}
16\pi^2\dot{\lambda} =& 6\lambda^2 - 3\lambda\left(3g_2^2 + \frac{3}{5}g_1^2\right) + 3g_2^4 + \frac{3}{2}\left(\frac{3}{5}g_1^2 + g_2^2\right)^2 + 4\lambda T \\
& - 8\operatorname{tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u Y_u^\dagger Y_u + 3Y_d^\dagger Y_d Y_d^\dagger Y_d\right) + 12\Lambda_4^2 + 8\Lambda_5^2 \qquad \text{(B5a)}\\
16\pi^2\dot{\Lambda}_1 =& -\frac{36}{5}g_1^2\Lambda_1 - 24g_2^2\Lambda_1 + \frac{108}{25}g_1^4 \underline{+18g_2^4} + \frac{72}{5}g_1^2g_2^2 \underline{+14\Lambda_1^2} + 4\Lambda_1\Lambda_2 + 2\Lambda_2^2 + 4\Lambda_4^2 + 4\Lambda_5^2 \\
& + 4\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Lambda_1 \underline{-8\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\Delta^\dagger Y_\Delta\right)} \qquad \text{(B5b)}\\
16\pi^2\dot{\Lambda}_2 =& -\frac{36}{5}g_1^2\Lambda_2 - 24g_2^2\Lambda_2 \underline{+12g_2^4} - \frac{144}{5}g_1^2g_2^2 + 3\Lambda_2^2 + 12\Lambda_1\Lambda_2 - 8\Lambda_5^2 + 4\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Lambda_2 \\
& + 8\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\Delta^\dagger Y_\Delta\right) \qquad \text{(B5c)}\\
16\pi^2\dot{\Lambda}_4 =& -\frac{9}{2}g_1^2\Lambda_4 - \frac{33}{2}g_2^2\Lambda_4 + \frac{27}{25}g_1^4 \underline{+6g_2^4} + \left[8\Lambda_1 + 2\Lambda_2 + 3\lambda + 4\Lambda_4 + 2T + 2\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\right]\Lambda_4 \\
& + 8\Lambda_5^2 - 4\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\nu^\dagger Y_\nu\right) \qquad \text{(B5d)}\\
16\pi^2\dot{\Lambda}_5 =& -\frac{9}{2}g_1^2\Lambda_5 - \frac{33}{2}g_2^2\Lambda_5 - \frac{18}{5}g_1^2g_2^2 + \left[2\Lambda_1 - 2\Lambda_2 + \lambda + 8\Lambda_4 \underline{+2T + 2\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta\right)}\right]\Lambda_5 \\
& + 4\operatorname{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\nu^\dagger Y_\nu\right)\ . \qquad \text{(B5e)}
\end{aligned}
$$

There is also a contribution to the gauge coupling renormalization due to the Higgs triplet

$$
\begin{aligned}
16\pi^2\beta_{g_1} =& \frac{41}{10}g_1^3 + \frac{1}{6}\cdot 3\cdot 2\frac{3}{5}g_1^3 = \frac{47}{10}g_1^3 \qquad \text{(B6)}\\
16\pi^2\beta_{g_2} =& -\frac{19}{6}g_2^3 + \frac{1}{6}\cdot 2\cdot 2g_2^3 = -\frac{5}{2}g_2^3\ , \qquad \text{(B7)}
\end{aligned}
$$

which has been also derived in [61]. The RG equation of the strong coupling constant remains unchanged

$$16\pi^2\beta_{g_3} = -7g_3^3\,. \tag{B8}$$

APPENDIX C: RG EQUATIONS IN THE MSSM

The RG equations which do not directly influence the renormalization of the operator generating neutrino masses are given by:

$$16\pi^2\dot{Y}_e = Y_e\left[3Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + \frac{3}{2}Y_\Delta^\dagger Y_\Delta\right] + Y_e\left[\mathrm{tr}(3Y_d^\dagger Y_d + Y_e^\dagger Y_e) + \frac{3}{2}|\Lambda_d|^2 - \frac{9}{5}g_1^2 - 3g_2^2\right] \tag{C1}$$

$$16\pi^2\dot{Y}_u = Y_u\left[Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right] + Y_u\left[\mathrm{tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) + \frac{3}{2}|\Lambda_u|^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right] \tag{C2}$$

$$16\pi^2\dot{Y}_d = Y_d\left[3Y_d^\dagger Y_d + Y_u^\dagger Y_u\right] + Y_d\left[\mathrm{tr}(3Y_d^\dagger Y_d + Y_e^\dagger Y_e) + \frac{3}{2}|\Lambda_d|^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right] \tag{C3}$$

$$16\pi^2\dot{\Lambda}_d = \Lambda_d\left[2\,\mathrm{tr}(3Y_d^\dagger Y_d + Y_e^\dagger Y_e) + \frac{7}{2}|\Lambda_d|^2 + \frac{1}{2}\,\mathrm{tr}(Y_\Delta^\dagger Y_\Delta) - \frac{9}{5}g_1^2 - 7g_2^2\right] \tag{C4}$$

There are also contributions to the evolution of the gauge couplings of electroweak interactions:

$$16\pi^2\beta_{g_1} = \frac{33}{5}g_1^3 + 2\cdot 3\frac{3}{5}g_1^3 = \frac{51}{5}g_1^3 \tag{C5}$$

$$16\pi^2\beta_{g_2} = 1g_2^3 + 2\cdot 2\,g_2^3 = 5g_2^3\,. \tag{C6}$$

The RG equation of the strong coupling constant remains unchanged

$$16\pi^2\beta_{g_3} = -3g_3^3\,. \tag{C7}$$

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[76] REAP/MixingParameterTools can be downloaded from `http://www.ph.tum.de/~rge`.

[77] The calculation of the necessary diagrams in the extended SM can be downloaded from `http://www.mpi-hd.mpg.de/~mschmidt/rgeTriplet`. Chao and Zhang checked the relevant parts of their calculation and obtained the same results as we do.

[78] The case without right–handed neutrinos has been studied in [70].

[79] We use GUT charge normalization: $\frac{3}{5}\left(g_1^{\mathrm{GUT}}\right)^2 = \left(g_1^{\mathrm{SM}}\right)^2$

[80] The terms coming from the Higgs triplet in our calculation agree with the calculation by Rossi [70].

[81] A Mathematica package with the exact formulae can be downloaded from `http://www.mpi-hd.mpg.de/~mschmidt/rgeTriplet/`.